Pion Form Factor: Status and Outlook

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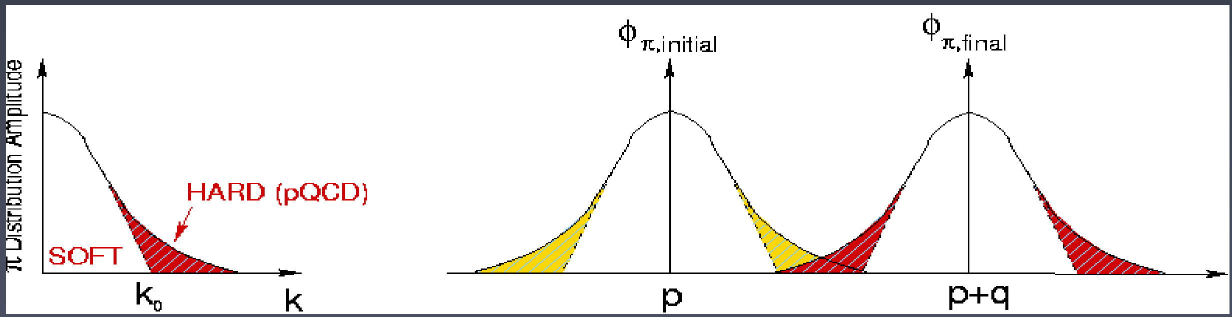
Meson Form Factors

Simple $q\bar{q}$ valence structure of mesons presents the ideal testing ground for our understanding of bound quark systems.

In quantum field theory, the form factor is the overlap integral:

$$F_\pi(Q^2) = \int \phi_\pi^*(p)\phi_\pi(p+q)dp$$



The meson wave function can be separated into φ_π^{soft} with only low momentum contributions ($k<k_0$) and a hard tail φ_π^{hard}.

While φ_π^{hard} can be treated in pQCD, φ_π^{soft} cannot.

From a theoretical standpoint, the study of the Q^2-dependence of the form factor focuses on finding a description for the hard and soft contributions of the meson wave-function.

QCD Hard Scattering Picture

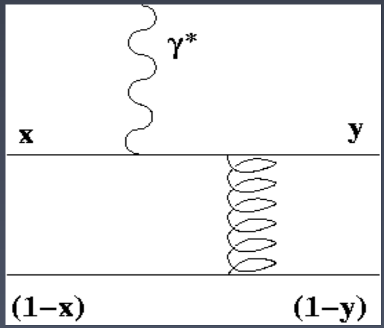
Example: π^+ ***Elastic Form Factor***

At large Q^2, perturbative QCD (pQCD) can be used

$$F_\pi(Q^2) = \frac{4\pi C_F \alpha_S(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log\left(\frac{Q^2}{\Lambda^2}\right)\right)^{-\gamma_n} \right|^2 \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q} \right) \right]$$

at asymptotically high Q^2, only the hardest portion of the wave function remains

$$\phi_\pi(x) \underset{Q^2\to\infty}{\to} \frac{3 f_\pi}{\sqrt{n_c}} x(1-x)$$



and F_π takes the very simple form

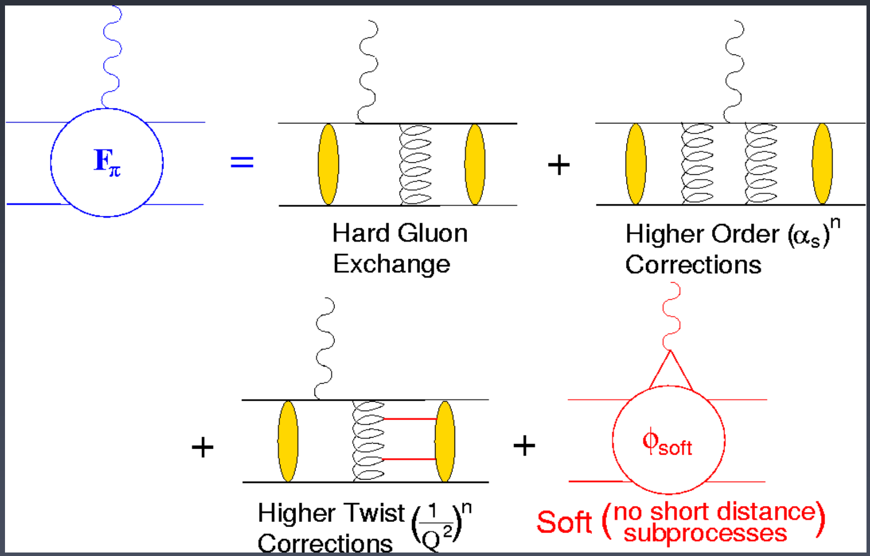
$$F_\pi(Q^2) \underset{Q^2\to\infty}{\to} \frac{16\pi\alpha_s(Q^2) f_\pi^2}{Q^2}$$

where f_π=92.4 MeV is the $\pi^+ \to \mu^+ \nu$ decay constant.

G.P. Lepage, S.J. Brodsky, Phys.Lett. **87B**(1979)359.

The Interplay of Hard and Soft Components

At experimentally-accessible Q^2, both the "hard" and "soft" components (e.g. transverse momentum effects) contribute.



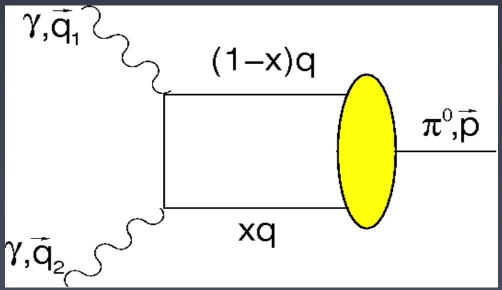
- **The interplay of hard and soft contributions is poorly understood.**
  - → Different theoretical viewpoints on whether higher-twist mechanisms dominate until very large momentum transfer or not.

- **The pion elastic and transition form factors experimentally accessible over a wide kinematic range.**
  - → A laboratory to study the **transition** from the soft to hard regime.

$\pi^0 \rightarrow \gamma^* \gamma$ Transition Form Factor

- Recent data from BaBar on the $\pi^0 \rightarrow \gamma^* \gamma$ transition form factor have only deepened the mystery on how QCD transitions from the hard to the soft regime.

- For real photons, $F_{\pi\gamma\gamma}$ determines rate of $\pi^0 \rightarrow \gamma\gamma$ decay, deeply related to axial anomaly.
- For virtual photons, since only one hadron is involved, $\pi^0 \rightarrow \gamma^* \gamma$ has the simplest structure for pQCD analysis.



In lowest order pQCD, the $\pi^0 \rightarrow \gamma^* \gamma$ transition form factor is given by

$$Q^2 F_{\pi\gamma\gamma}(Q^2) = \frac{2 f_\pi}{3} \int_0^1 \frac{dx}{x} \phi_\pi(x, Q^2) + O(\alpha_s) + O\left(\frac{\Lambda^2_{QCD}}{Q^2} \right)$$

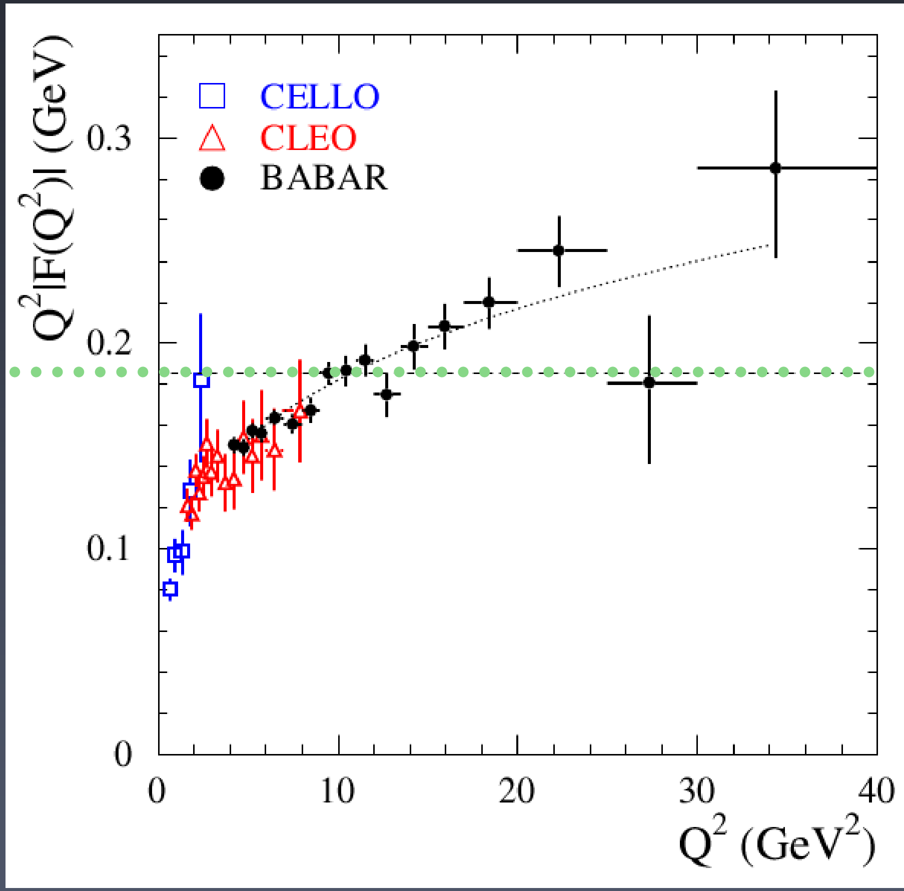
where the pion decay constant $f_\pi = 92.4$ MeV.

$F_{\gamma^*\gamma\pi^0}(Q^2)$ Experimental Results

- Unlike older data, new BaBar results exceed the pQCD asymptotic limit

$$F_{\gamma^*\gamma\pi^0}(Q^2) \underset{Q^2 \to \infty}{\to} \frac{2 f_\pi^2}{Q^2}$$

- The main problem, theoretically, is that the data show no tendency to level off at a particular value.
- Because of its potential ramifications, I will briefly discuss how the result was obtained.

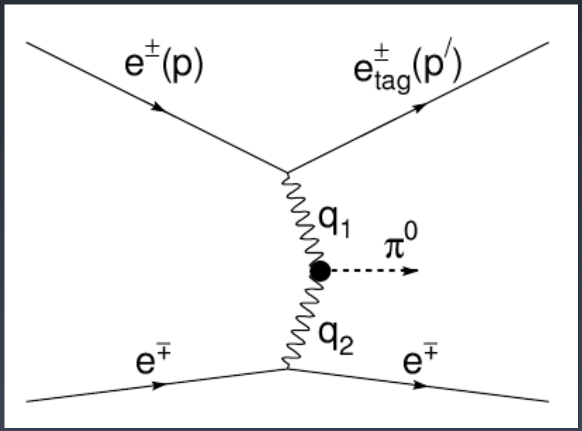


CELLO *[Z.Phys.* ***C49*** *(1991) 401]*

CLEO *[Phys. Rev.* ***D57*** *(1998) 33]*

BABAR *[Phys. Rev.* ***D80*** *(2009) 052002]*

BaBar $\pi^0 \to \gamma^* \gamma$ Data *[B. Aubert et al., PRD **80**(2009)052002]*



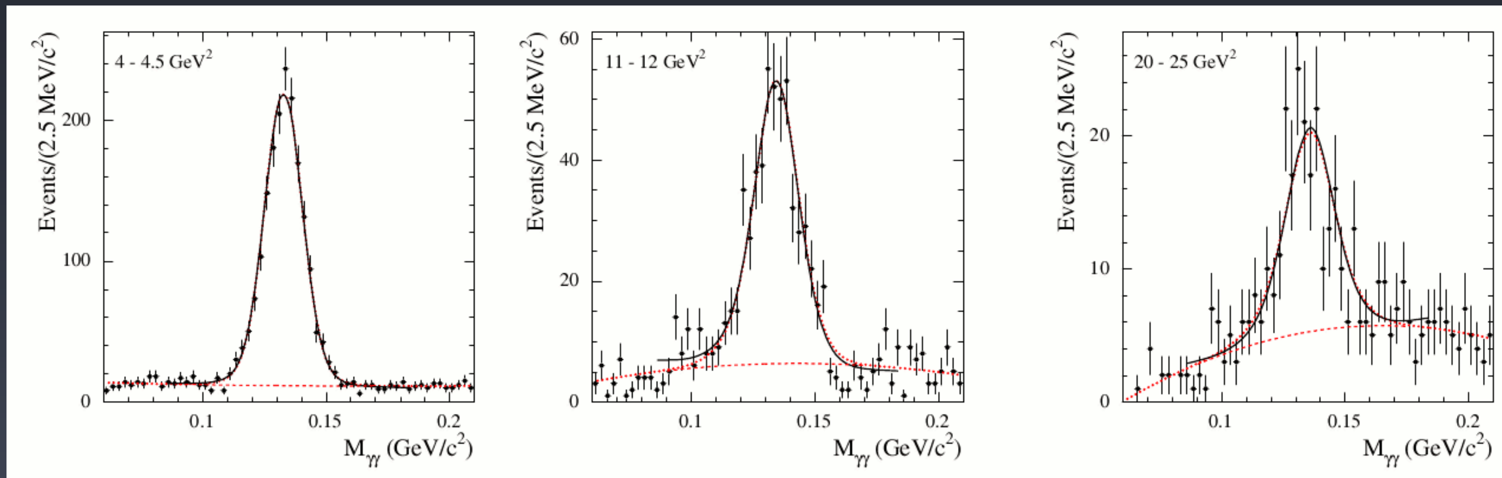
$e^+e^- \to e^+e^-\pi^0$ reaction utilized for space-like π^0 production.

- One $e^{\pm}$ scattered at large angle (detected) yielding virtual photon with large Q^2.
- Second $e^{\pm}$ (undetected) scattered at small angle yielding "nearly real" photon.
- 442 fb^{-1} data at 10.6 GeV CM energy used in analysis.

Data analysis is challenging:

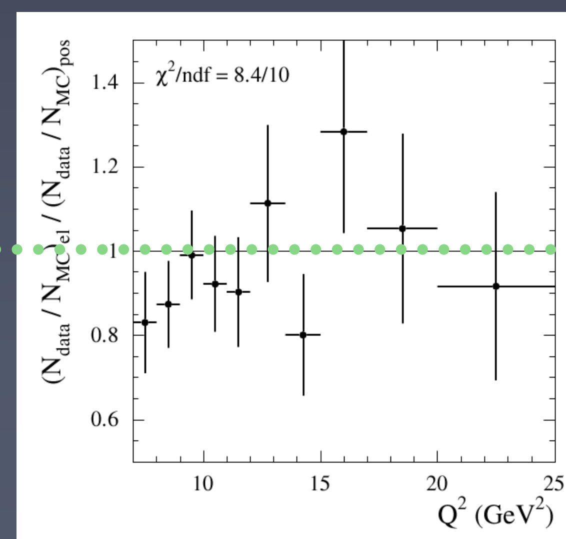
- Dominant background, Virtual Compton Scattering, exceeds π^0 production cross section by $>10^3$.
- Because of high π^0 lab frame energy, most $\pi^0 \to \gamma\gamma$ decays resolved as only single γ cluster in calorimeter.
- Virtual Compton data filter has 50-80% efficiency for signal events (depending on π^0 energy), which are then searched for $e^{\pm}$ and π^0 candidate.

BaBar $\pi^0 \to \gamma^*\gamma$ Data *[B. Aubert et al., PRD **80**(2009)052002]*



2γ invariant mass spectra for three Q^2 intervals.

- Highest Q^2=25-30GeV^2, 30-40 GeV^2 bins have <50 π^0 events each.
- No π^0 signal above background for Q^2>40 GeV^2.



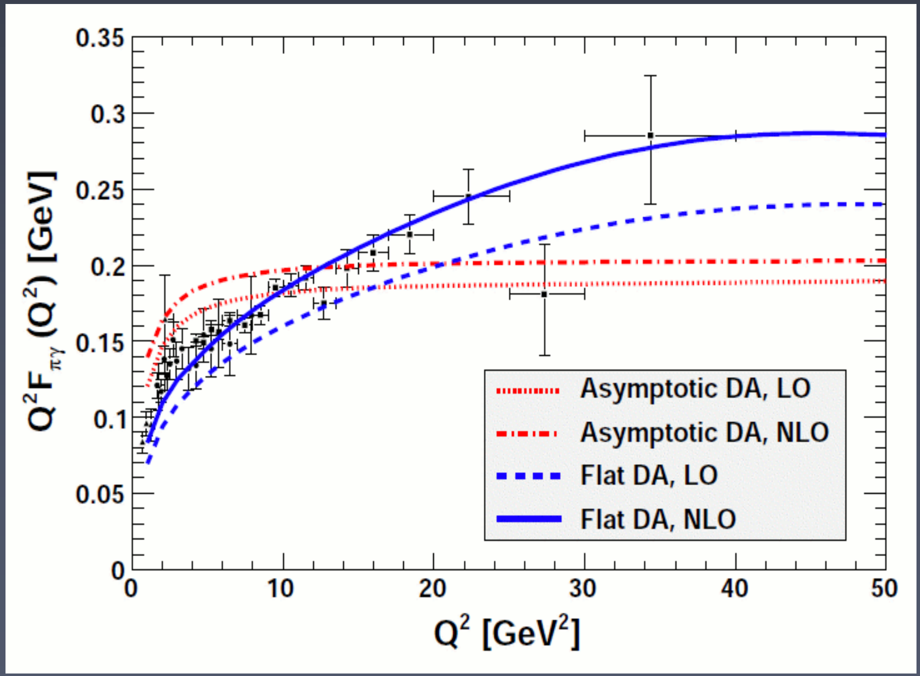
Data analysis check:

- Asymmetric beams (3.1GeV e$^+$, 9GeV e$^-$) produce very different energy, angular distributions, and trigger corrections, for e$^-$ tag and e$^+$ tag events.
- Differential cross section comparison for e$^-$, e$^+$ tagged events should be unity.

What do the $\pi^0 \rightarrow \gamma^*\gamma$ data imply?

- Non-perturbative information about pion is accumulated in the pion distribution amplitude $\varphi_\pi(x)$.
  → Comparison of pQCD predictions with data gives information on the shape of $\varphi_\pi(x)$.

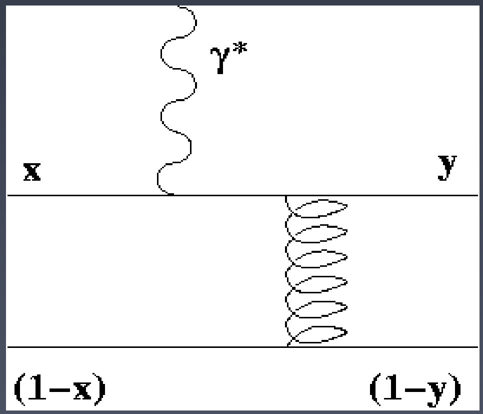
- Various authors have indicated that BaBar data consistent with "flat" DA.
  → enhanced contributions at high and low x.
- A flat DA was not anticipated.



Li and Mishima, PRD **80**(2009)074024.

Implications

- If the BaBar data are correct, they have broad implications.
  - e.g. does GPD factorization apply at experimentally accessible Q^2?
  - It is important to see if other "hard" exclusive processes involving the pion behave in a consistent fashion.
- F_π: Radyushkin [PRD**80**(2009)094009] finds that if the "flat DA scenario" holds, the one gluon-exchange diagram should contribute negligibly.
  - The gluon line should be absorbed into the soft wave function, and the pion form factor should be calculated non-perturbatively.



- High quality $Q^2F_\pi(Q^2)$ data are clearly required to delineate the role of hard versus soft contributions in the pion DA.

  → the acquisition of these data have their own challenges.

Determination of F_π via Pion Electroproduction

At low Q^2<0.3 GeV², the π^+ form factor can be measured **exactly** using high energy π^+ scattering from atomic electrons.

⇒ 300 GeV pions at CERN SPS. *[Amendolia et al., NP **B277**(1986)168]*

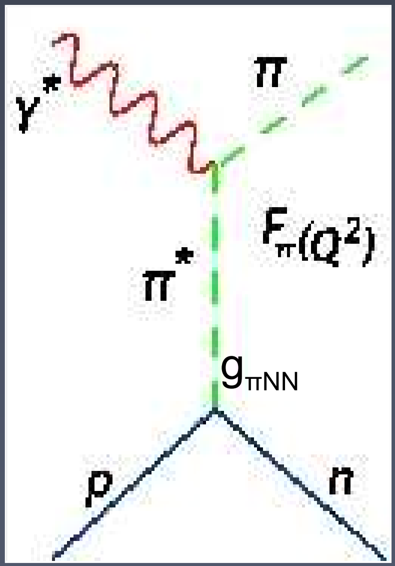
⇒ Provides an accurate measure of the π^+ charge radius.

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

To access higher Q^2, one must employ the $p(e,e'\pi^+)n$ reaction.

- t-channel process dominates σ_L at small $-t$.
- In the Born term model:

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} g_{\pi NN}^2(t)\, F_\pi^2(Q^2,t)$$



Chew-Low Method to determine Pion Form Factor

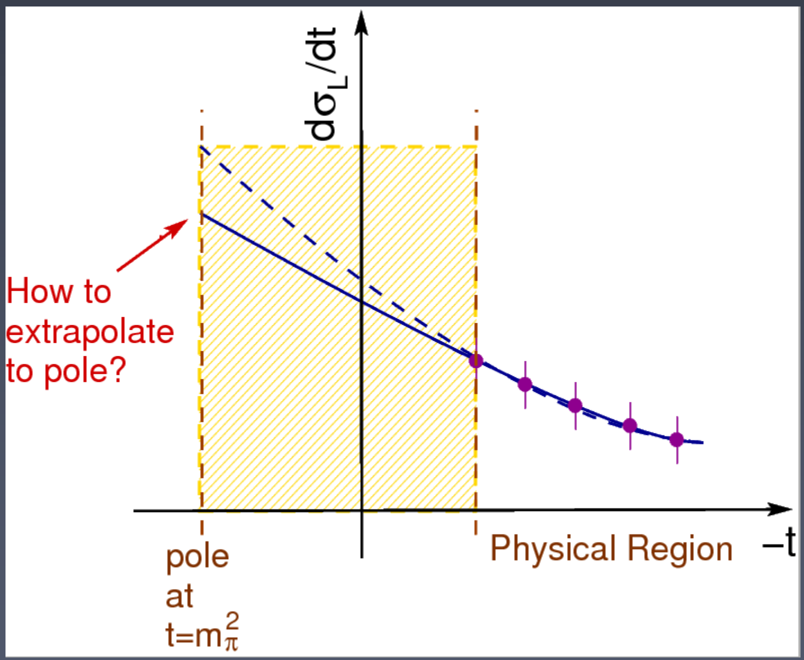
p(e,e'π⁺)n **data are obtained some distance from the** $t=m_\pi^2$ **pole.**

→ "Chew Low" extrapolation method requires knowing the analytic dependence of $d\sigma_L/dt$ **through the unphysical region.**

Extrapolation method last used in 1972 by Devnish & Lyth [PRD **5**,47].

- Very large systematic uncertainties.
- Failed to produce reliable result.

  → Different polynomial fits equally likely in physical region gave divergent form factor values when extrapolated to $t=m_\pi^2$**.**



The Chew-Low Method was subsequently abandoned.

Only reliable approach is to use a model incorporating the π^+ production mechanism and the `spectator' nucleon to **extract** F_π from σ_L.

- JLab F_π experiments use the Vanderhaeghen-Guidal-Laget (VGL) Regge model as it has proven to give a reliable description of σ_L across a wide kinematic domain.
  *[Vanderhaeghen, Guidal, Laget, PRC **57**(1998)1454]*
- More models would allow a better understanding of the model dependence of the F_π result. There has been considerable recent interest:
  - *M.M. Kaskulov, U. Mosel, PRD **81**(2010)045202.*
  - *S.V. Goloskokov, P. Kroll, Eur.Phys.J. **C65**(2010)137.*
  - *C. Bechler, D. Müller, arXiv:0906.2571 [hep-ph].*
  - *A. Faessler, T. Gutsche, V.E. Lyubovitskij, I.T. Obukhovsky, PRC **76**(2007)025213.*

Our philosophy remains to publish our experimentally measured $d\sigma_L/dt$, so that updated values of $F_\pi(Q^2)$ can be extracted as better models become available.

Extract $F_\pi(Q^2)$ from σ_L data via VGL Regge Model

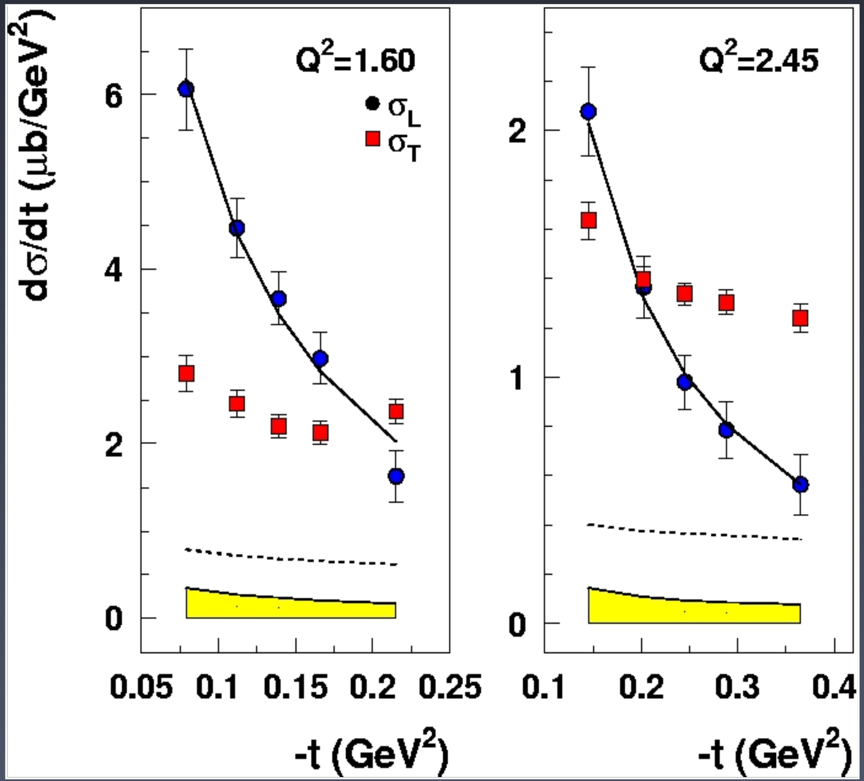
- Feynman propagator $\left(\frac{1}{t - m_\pi^2}\right)$

  replaced by π and ρ Regge propagators.
  - Represents the exchange of a series of particles, compared to a single particle.
- Model parameters fixed from pion photoproduction.
- Free parameters: Λ_π, Λ_ρ (trajectory cutoff).

  [Vanderhaeghen, Guidal, Laget, PRC **57**(1998)1454]

$$F_\pi = \frac{1}{1 + Q^2/\Lambda_\pi^2}$$

Fit to σ_L to model gives F_π at each Q^2.



Fπ-2 data: T. Horn et al., PRL **97**(2006)192001.

Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature.
Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

Λ_π^2=0.513, 0.491 GeV^2, Λ_ρ^2=1.7 GeV^2.

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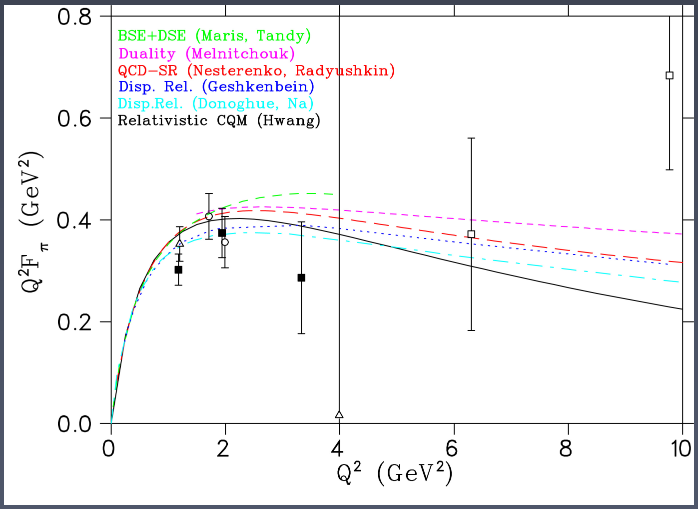
Issues with High Q^2 Cornell data

Q^2 (GeV^2)	W (GeV)	$-t_{min}$ (GeV^2)	Comments
1.20 3.99	3.08 2.14	0.019 0.477	High ε unsep σ only. π^-/π^+ data on ^2H used for isoscalar correction to unsep dσ/dt. PRD **13**(1976)25.
1.71 1.99	3.09 2.14	0.034 0.157	High ε unsep σ only. π^-/π^+ isoscalar correction from other ^2H kinematics used. PRD **13**(1976)25.
1.18 1.94 3.33	2.11 2.67 2.63	0.069 0.070 0.162	High and low ε from different expts used. Systematic error? PRL **37**(1976)1326.
6.30 9.77	2.66 2.63	0.43 0.87	Low ε unsep σ only. t-channel Born Term model used to extract F_π. Uncontrolled systematic errors! PRD **17**(1978)1693.

- **Problematic L/T separation.**
  - High and low ε from different expts used, or only low ε setting taken.
  - In all cases, a model for σ_T was used when extracting σ_L and F_π.
- **Analysis based on assumptions with difficult to quantify systematic errors.**
  - Data taken far from pole, with $-t_{min}$ *as high as 40* m_π^2.



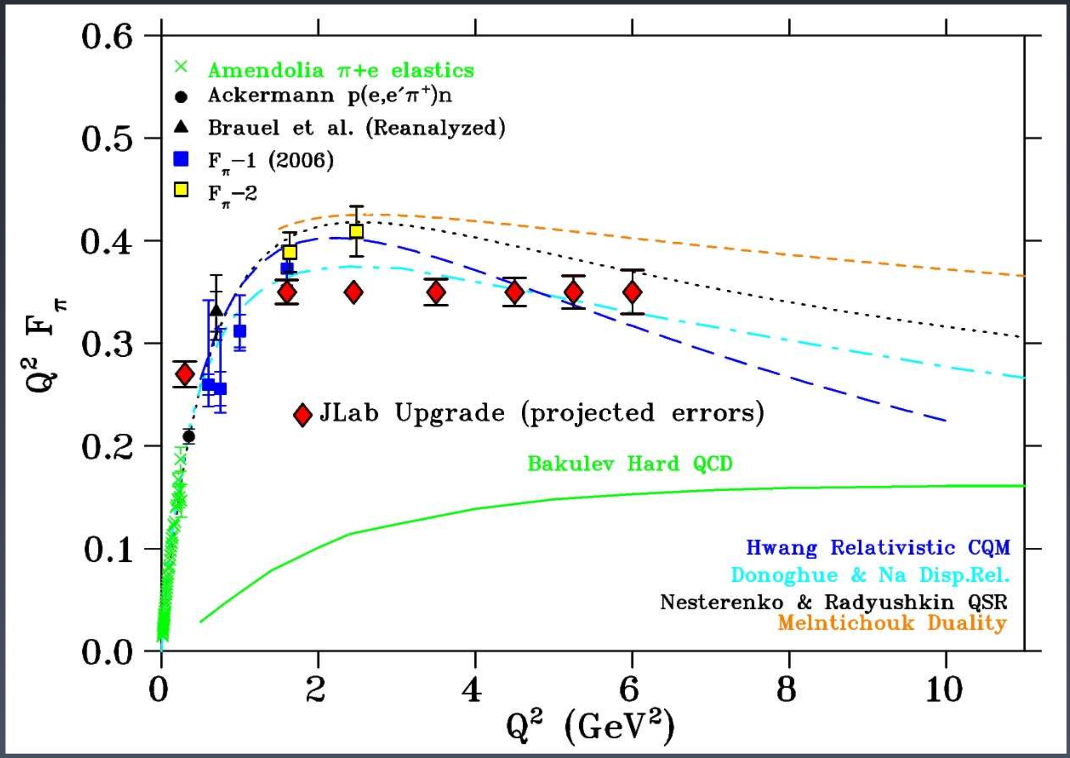
"[we] question whether F_π has been truly determined for large Q^2."

C.E. Carlson, J. Milana, PRL **65**(1990)1717.

Reliable F_π results require appropriately-chosen kinematics

- Experiment must access small $-t$ to ensure t-channel dominance.
- Carlson and Milana [PRL **65**(1990)1717] looked at competing non-pole QCD processes complicating the extraction of F_π at large Q^2.
  - background ratio M_{pQCD}/M_{pole} rises dramatically once $-t_{min}>0.20$.
  - "more reliable measurements of F_π at high Q^2 require smaller $|t|$ and thus higher electron energy loss ν."
- **Note:** if theoretical calculations relating to the background processes were available, we could possibly extend the useful $-t$ range for F_π measurements, resulting in higher Q^2 data.

Expected F_π Measurements with JLab Upgrade



E12-06-101:
G. Huber and
D. Gaskell
spokespersons

- 10.9 GeV electron beam and SHMS θ=5.5^o capability will allow F_π to be determined up to Q^2=6.0 GeV^2.
  - Slightly lower than Q^2=7.6 GeV^2 theoretical upper limit for E_{beam}=10.9 GeV, $-t_{min}$<0.21 GeV^2 due to $\Delta\varepsilon$>0.25 needed for reliable L/T separation.
- Approved with "A" priority in January, 2010.

Model/Intepretation Issues

- A common criticism of the electroproduction technique is the difficulty to be certain one is measuring the "physical" form factor.

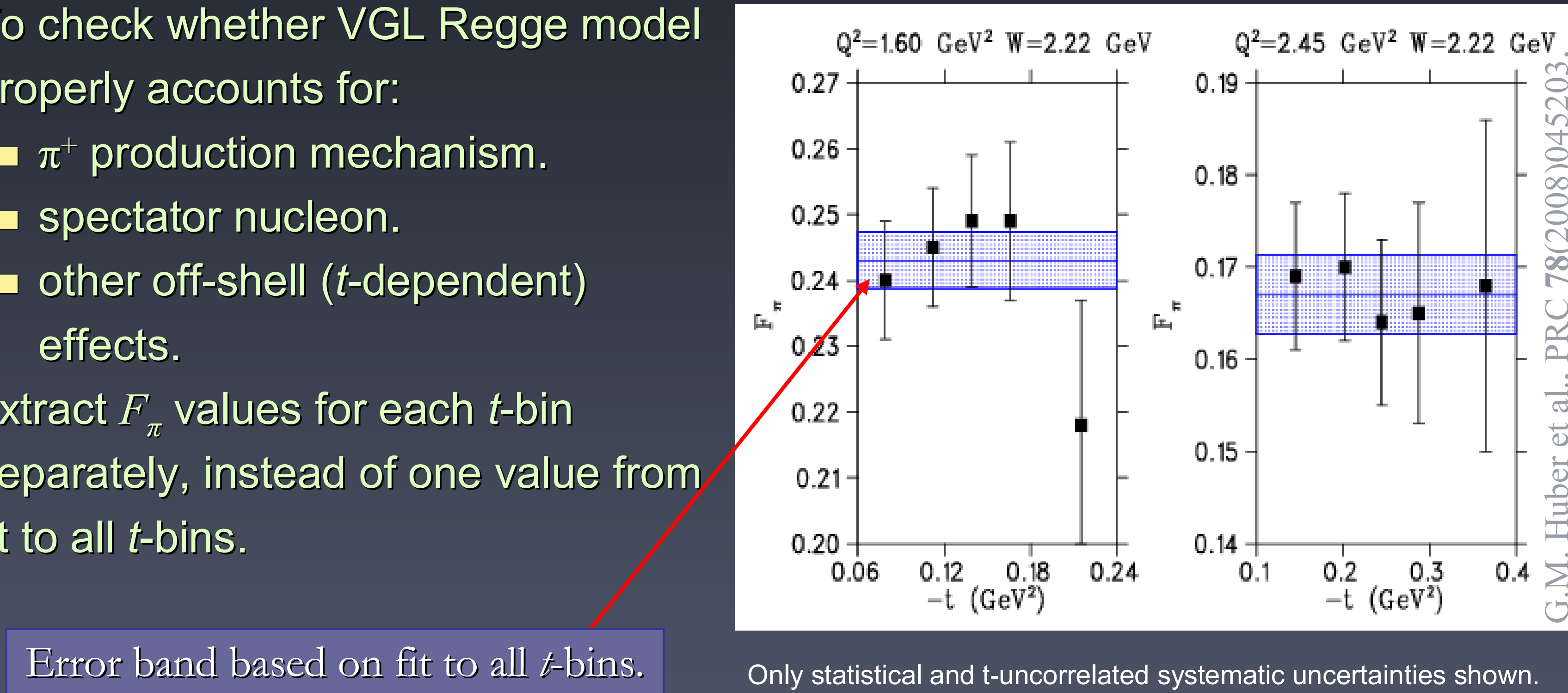
> "What is at best measured in electroproduction is the transition amplitude between a mesonic state with an effective space-like mass $m^2=t<0$ and the physical pion. It is theoretically possible that the off-shell form factor $F_\pi(Q^2,t)$ is significantly larger than the physical form factor because of its bias towards more point-like $q\bar{q}$ valence configurations within its Fock state structure." --S.J. Brodsky, Handbook of QCD, 2001.

- What tests/studies can we do to give confidence in the result?
  - Check consistency of model with data.
  - Extract form factor at several values of $-t_{min}$ for fixed Q^2.
  - Test that the pole diagram is really the dominant contribution to the reaction meachnism.
  - Verify that electroproduction technique yields results consistent with π-e elastic scattering at same Q^2.

VGL $p(e,e'\pi^+)n$ model check

- To check whether VGL Regge model properly accounts for:
  - π^+ production mechanism.
  - spectator nucleon.
  - other off-shell (*t*-dependent) effects.

  extract F_π values for each *t*-bin separately, instead of one value from fit to all *t*-bins.



Only statistical and t-uncorrelated systematic uncertainties shown.

- Deficiencies in model may show up as *t*-dependence in extracted $F_\pi(Q^2)$ values.
- Resulting F_π values are insensitive (<2%) to *t*-bin used.
- Lends confidence in applicability of VGL model to the kinematical regime of the JLab data, and the validity of the extracted $F_\pi(Q^2)$ values.

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Form Factor Extraction at different $-t_{min}$

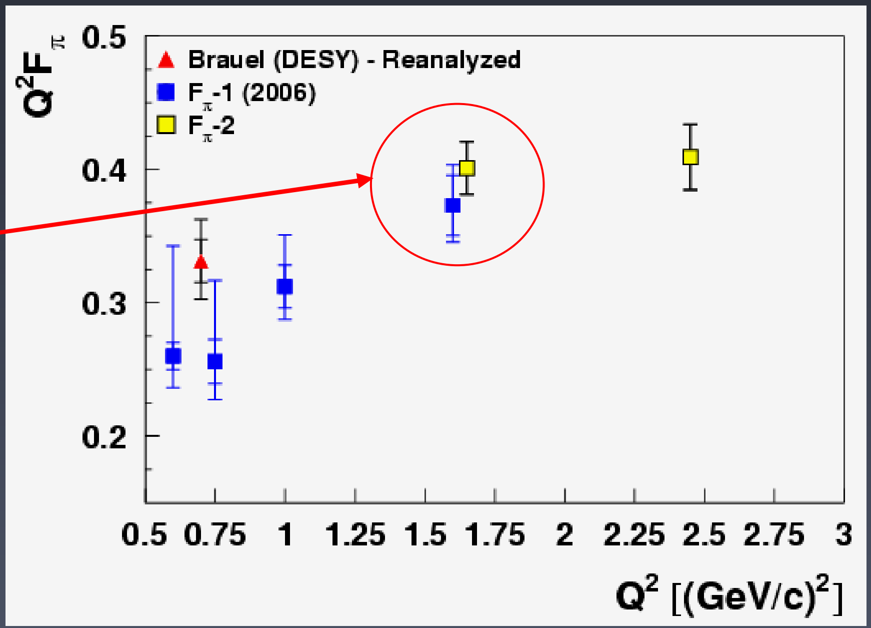
Does the VGL model handle the "off-shellness" of the pion appropriately?

Test by extracting F_π at different distances from pole.

Expt: F_π-2, $-t_{min}=0.093\ GeV^2$
$W=2.22\ GeV$.
F_π-1, $-t_{min}=0.15\ GeV^2$
$W=1.95\ GeV$.

***W=2.22* point 30% closer to pole.**

→ Agreement ~4%.



Additional data after 12 GeV upgrade will allow further tests:

$Q^2=1.6\ GeV^2$ $-t_{min}=0.029\ GeV^2$, $W=3.00\ GeV$.

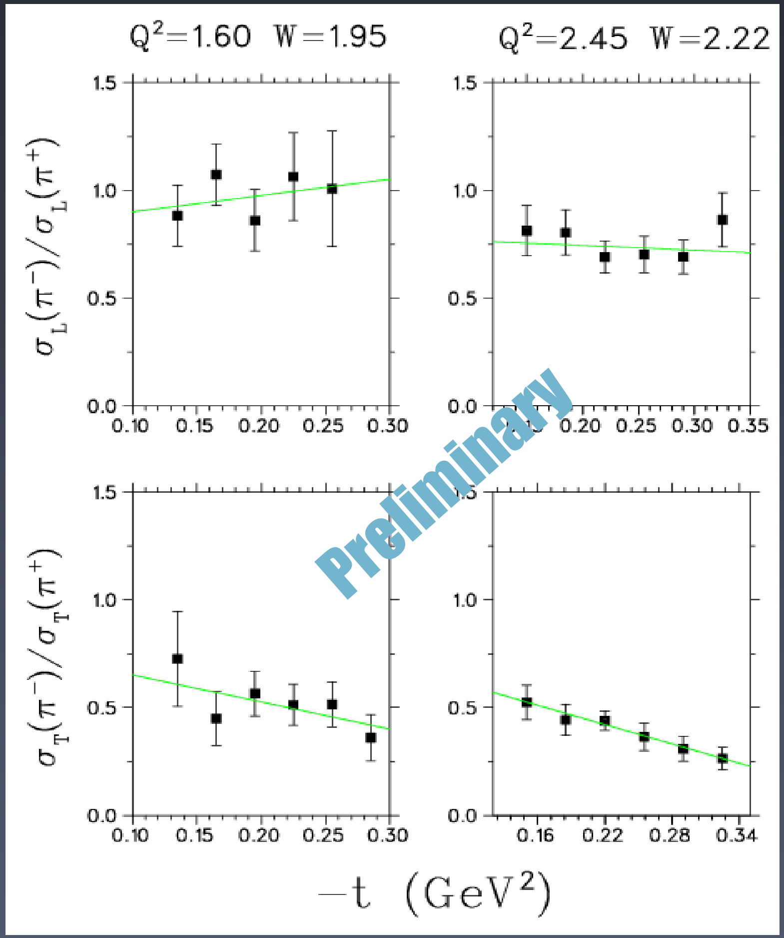
$Q^2=2.45\ GeV^2$ $-t_{min}=0.048\ GeV^2$, $W=3.20\ GeV$.

Check that σ_L is dominated by π *t*-channel process

- ^{2}H target L/T separations.
- π^+ *t*-channel diagram is purely isovector (G-parity conservation).

$$R_L = \frac{\sigma_L[n(e,e'\pi^-)p]}{\sigma_L[p(e,e'\pi^+)n]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}$$

- Isoscalar backgrounds (such as $b_1(1235)$ contributions to *t*-channel) will dilute ratio.



Error bars indicate statistical and estimated random (pt-pt) systematic uncertainties in quadrature.
Correlated (scale) and partly correlated (t-corr) systematic uncertainties are not yet added.

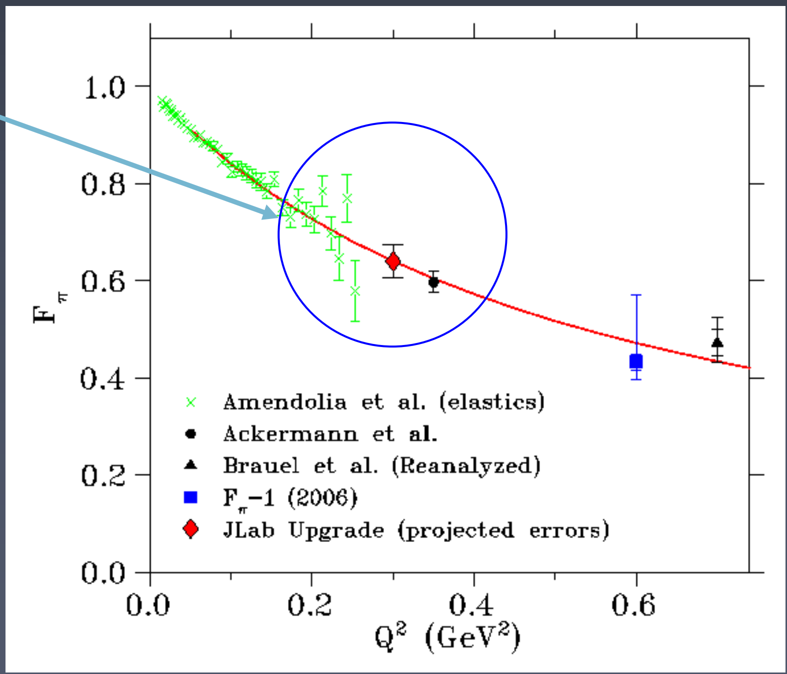
- R_L values consistent with pion-pole dominance ($|A_S/A_V|$<5%).
- Further tests planned in E12-06-101 at Q^2=1.60 and 3.50 GeV^2.

Electroproduction Method Check

Directly compare $F_\pi(Q^2)$ values extracted from very low $-t$ electroproduction with the exact values measured in elastic e-π scattering.

<u>METHOD PASSES CHECKS:</u>

- Q^2=0.35 GeV^2 data from DESY consistent with limit of elastic scattering data within uncertainties.
  *[H. Ackermann, et al., NP **B137**(1978)294]*
- A much better check is planned in E12-06-101 by taking Q^2=0.30 GeV^2 data at 50% lower $-t$ (0.005 GeV^2).



Summary

- Access to meson form factors in space-like region experimentally difficult:
  - π^0 measurements most direct, but data analysis challenging.
  - π^+ requires model to extract FF at physical meson mass.
- Old $F_{\gamma^*\gamma\pi^0}(Q^2)$ data suggested perturbative behavior at $Q^2 \approx 1$ GeV^2, but new BaBar data indicate perturbative region could be very far away.
  - It seems essential to probe all exclusive π production channels to obtain a consistent/global understanding.
- F_{π^+} results at $Q^2=2.5$ GeV^2 also indicate that soft processes seem to play an important role for π^+.
- JLab 12 GeV upgrade will dramatically improve π^+ data set.
  - Many checks planned to determine reliability of result.
- Where is the transition to the perturbative regime?